LPL Laboratoire de Physique des Lasers

Turbulence in superfluids: waves and vortices

Romain Dubessy & BEC group

Laboratoire de physique des lasers, CNRS UMR 7538
Université Sorbonne Paris Nord, Villetaneuse, France

Physics of Wave Turbulence and beyond

Outline

1) introduce *dilute* atomic **superfluids**

basic properties, experiments, measurements, orders of magnitude

2) show some experimental results

equilibrium, linear dynamics, nonlinear

3) explain what we do at LPL

and why it may be relevant for superfluid turbulence

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Disclaimer: what I will not do

- show pictures of Sergey
- write wake kinetic equations
- cite all litterature

(and I apologize for that)

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Wave turbulence and vortices in Bose–Einstein condensation

Sergey Nazarenko[a], Miguel Onorato[b,*]

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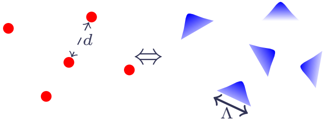
Communicated by A.C. Newell

What is a Bose-Einstein condensate ?

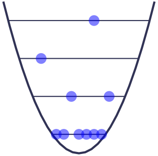
Quantum mechanics:
particles behave as waves.

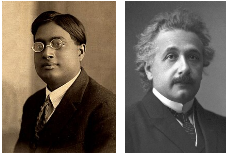


De Broglie (1923)

Quantum statistics:
bosons accumulate in
the lowest energy level

$$n\Lambda^3 > 2.61$$





Bose & Einstein (1925)

$$n \sim \frac{1}{d^3} \qquad \Lambda = \frac{h}{\sqrt{2\pi M k_B T}}$$

Need a trap and advanced cooling techniques

What is a Bose-Einstein condensate ?

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Quantum mechanics:
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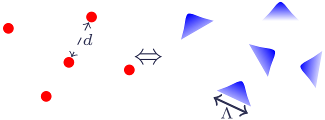

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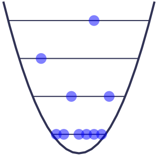
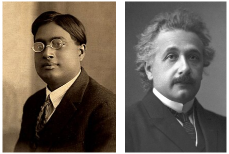




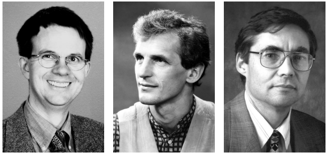
Bose & Einstein (1925)

$$n \sim \frac{1}{d^3} \qquad \Lambda = \frac{h}{\sqrt{2\pi M k_B T}}$$

Need a trap and advanced cooling techniques

- ⇒ laser cooling
- ⇒ magnetic / optical traps
- ⇒ evaporation

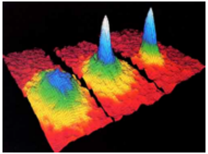
Chu, Cohen-Tannoudji & Phillips (Nobel 1997)



Cornell, Ketterle & Wieman (Nobel 2001)

First BEC achieved in 1995
Sodium & Rubidium

Since then: Li, H, He, K, Cs, Cr, Yb, Ca, Sr, Dy, Er, molecules, ...



Superfluid properties ?

Quantum fluids are superfluids:

- $\Rightarrow$ no viscosity — *critical velocity* v_c
- $\Rightarrow$ irrotational flow — *quantum vortices*

$$i\hbar\frac{\partial\psi}{\partial t} = \left(-\frac{\hbar^2\boldsymbol{\nabla}^2}{2M} + g|\psi|^2 + V(\boldsymbol{r},t) - \mu\right)\psi$$

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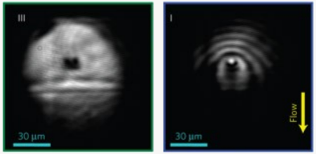
Classical fluid

Wikimedia: wake of a boat

Quantum fluid

$v < v_c$ $v > v_c$

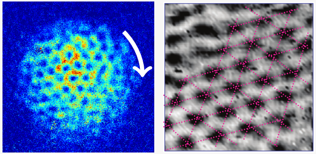


[Amo et al., Nat. Phys. (2009)]

No wake for $v < v_c$!

(generally: $v_c \leq$ sound)

Vortex lattices:



rotating superfluid, LPL

superconductor

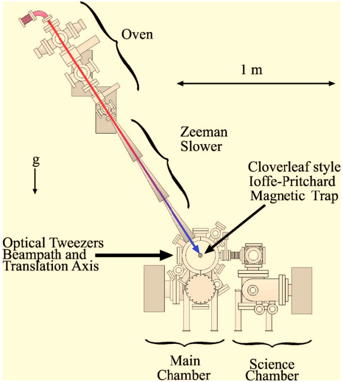
[Guillamón et al., Nat. Phys. (2009)]

Superfluid flow without dissipation

Rotating superfluid is analogous to a superconductor

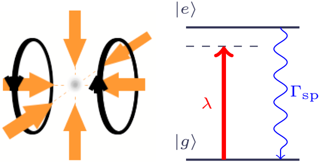
How to produce a ultra-cold gaz ?

Path: MOT $\Rightarrow$ optical molasses $\Rightarrow$ conservative trap $\Rightarrow$ evaporative cooling to BEC



MIT, 2006, Rubidium

[Ketterle et al., *Making, probing and understanding BECs* (1999)]



Stage	n (cm^{-3})	v or T	PSD$= n\Lambda^3$
Oven @400 K	10^{13}	350 m/s	10^{-14}
Slowed beam	10^{7}	40 m/s	10^{-18}
MOT	10^{10}	150 μK	10^{-7}
c-MOT	10^{11}	300 μK	4×10^{-7}
Molasses	10^{11}	10 μK	6×10^{-5}
Magnetic trap	10^{11}	500 μK	2×10^{-7}
BEC	3×10^{13}	<500 nK	2.61

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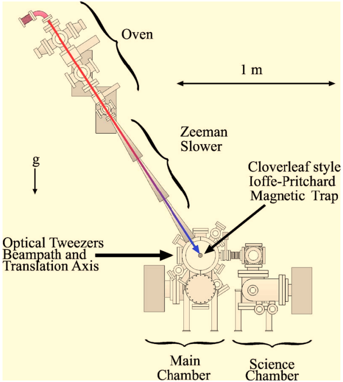

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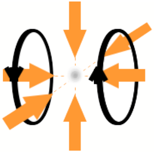

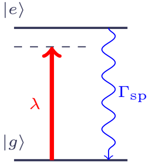

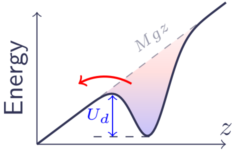

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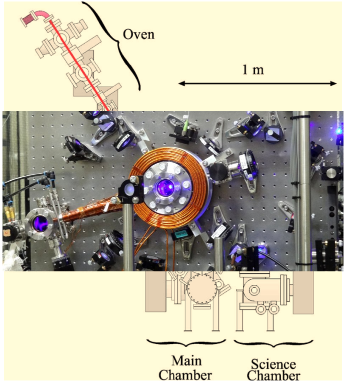

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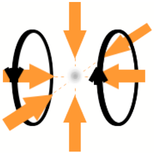

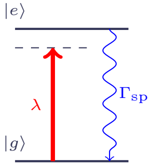

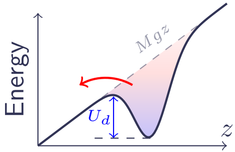

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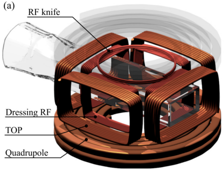
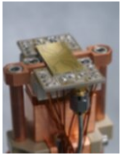
Examples of traps for ultra-cold atoms

Magnetic traps

$$V(\boldsymbol{r}) = -\langle \hat{\boldsymbol{\mu}} \cdot \boldsymbol{B}(\boldsymbol{r}) \rangle \propto |\boldsymbol{B}(\boldsymbol{r})|$$

$\Rightarrow$ trap near a local minimum of B-field





$$V(\boldsymbol{r}) = \frac{M}{2}\left(\omega_x^2 x^2 + \omega_y^2 y^2 + \omega_z z^2\right)$$

pros

- $\Rightarrow$ easy & stable
- $\Rightarrow$ extremely smooth

cons

- $\Rightarrow$ large scale only
- $\Rightarrow$ bulky

[Oxford, Paris, ...]

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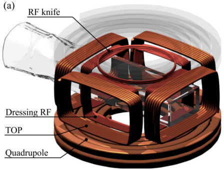

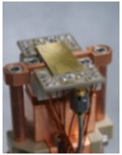

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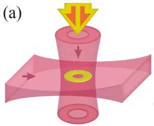
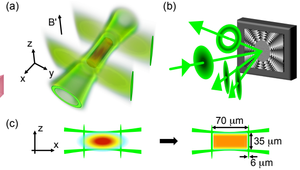
[Oxford, Paris, ...]

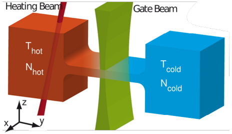
Optical traps

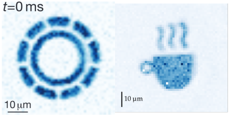
$$V(\boldsymbol{r}) = -\langle \hat{\boldsymbol{d}} \cdot \boldsymbol{E}(\boldsymbol{r}) \rangle \propto I(\boldsymbol{r})$$

$\Rightarrow$ trap near local extrema of intensity









pros

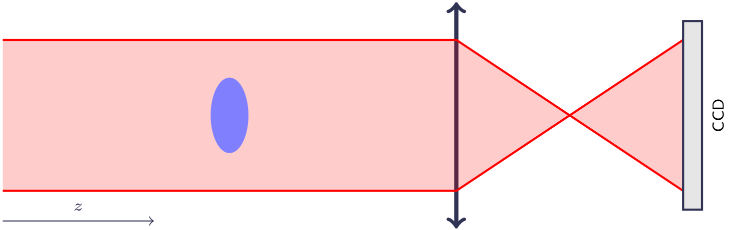
- $\Rightarrow$ highly versatile
- $\Rightarrow$ scalable

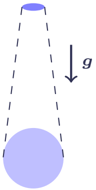
cons

- $\Rightarrow$ heating & stability
- $\Rightarrow$ rugosity

[NIST, Cambridge, Zurich, Paris, ...]

How to probe a quantum gaz ?

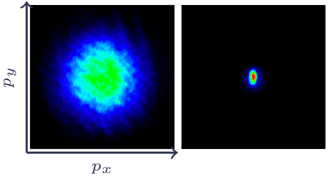




Record the *shadow* of the cloud on the camera.

⇒ in situ $\quad n_{2D}(x, y) = \int dz\, n(\boldsymbol{r})$

⇒ after a time-of-flight
mapping: $\boldsymbol{r} = \boldsymbol{r}_0 + \boldsymbol{v}t \Rightarrow$ access to momentum distribution.



Thermal gaz: $\Delta p_x = \Delta p_y = \sqrt{M k_B T}$

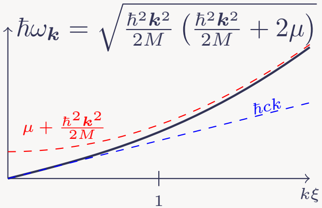
Condensate: $\Delta p_x = \frac{\hbar}{2\Delta x} \neq \Delta p_y$

Orders of magnitude

$$i\hbar\frac{\partial\psi}{\partial t} = \left(-\frac{\hbar^2\boldsymbol{\nabla}^2}{2M} + g|\psi|^2 + V(\boldsymbol{r},t) - \mu\right)\psi, \qquad N = \int d\boldsymbol{r}\,|\psi|^2, \qquad g = \frac{4\pi\hbar^2}{M}a_s$$

two-body scattering length: a_s
healing length: $\xi = \frac{\hbar}{\sqrt{2M\mu}}$
speed of sound: $c = \sqrt{\frac{\mu}{M}}$

homogeneous system $V(\boldsymbol{r},t) = 0$



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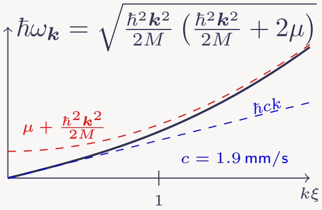
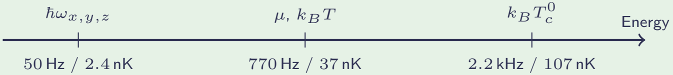


Harmonic trap

$V(\boldsymbol{r}) = \frac{M}{2}\left(\omega_x^2 x^2 + \omega_y^2 y^2 + \omega_z^2 z^2\right)$

Take $N = 10^5$ ^{87}Rb atoms in a isotropic trap $M = 1.44 \times 10^{-25}$ kg



$\Rightarrow n(\boldsymbol{r}) = |\psi|^2 = \frac{\mu - V(\boldsymbol{r})}{g}$ Thomas-Fermi inverted parabola

Orders of magnitude

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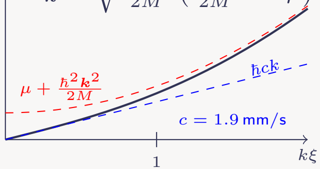
homogeneous system

$V(\boldsymbol{r},t) = 0$

$$\hbar\omega_{\boldsymbol{k}} = \sqrt{\frac{\hbar^2\boldsymbol{k}^2}{2M}\left(\frac{\hbar^2\boldsymbol{k}^2}{2M} + 2\mu\right)}$$



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Energy	$\hbar\omega_{x,y,z}$	$\mu,\ k_BT$	$k_BT_c^0$
	50 Hz / 2.4 nK	770 Hz / 37 nK	2.2 kHz / 107 nK

$\Rightarrow n(\boldsymbol{r}) = |\psi|^2 = \frac{\mu - V(\boldsymbol{r})}{g}$ Thomas-Fermi inverted parabola

length	a_s	ξ	a_ω	R_{TF}
	5 nm	0.3 μm	1.5 μm	8.5 μm

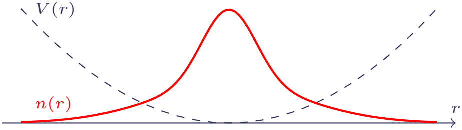
For an anisotropic trap $\omega_{x,y} \ll \omega_z$, if $\boxed{\hbar\omega_{x,y} \ll \mu,\ k_BT \leq \hbar\omega_z}$

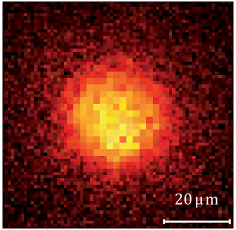
z degree of freedom is frozen $\Rightarrow$ **2D dynamics**

Example: measurement of the equation of state of a many-body system.

- $\Rightarrow$ in-situ density profile $n(\boldsymbol{r})$
- $\Rightarrow$ knowledge of the trap $V(\boldsymbol{r})$
- $\Rightarrow$ local density approximation

$$\mu_{\rm loc}(\boldsymbol{r}) = \mu_0 - V(\boldsymbol{r})$$





[Desbuquois et al., PRL (2014)]

What can we do with a superfluid ? (I)

Equilibrium properties

Example: measurement of the equation of state of a many-body system.

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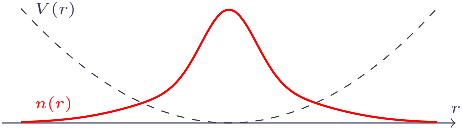

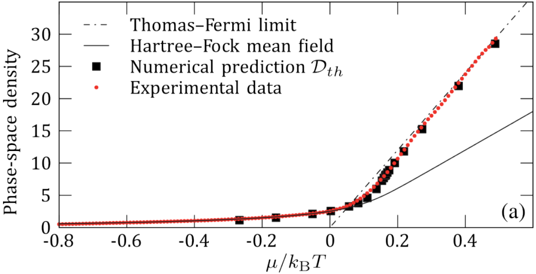

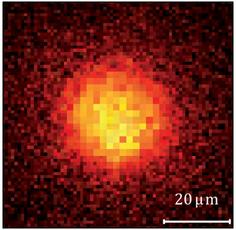

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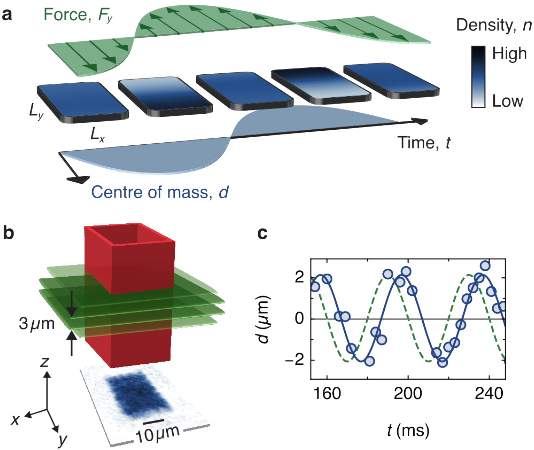
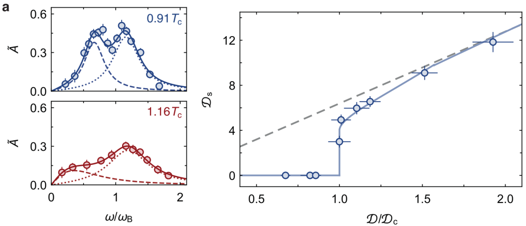


[Christodoulou et al., Nature (2021)]

⇒ apply weak oscillating force to excite sound waves

⇒ record center-of-mass oscillations

⇒ vary drive frequency: spectroscopy



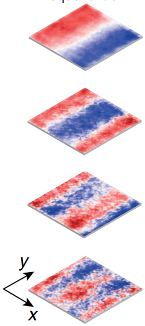
⇒ two peaks ⇔ two speeds of sound

⇒ measure of the superfluid fraction

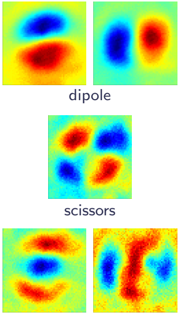
What can we do with a superfluid ? (III)
Wave propagation

Small amplitude waves

2D square box



2D harmonic



dipole

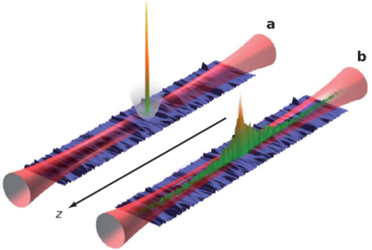
scissors

quadrupole, monopole

⇒ dispersion relation

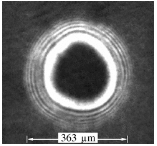
[Galka et al., PRL (2022), Dubessy et al., NJP (2014)]

Anderson localization



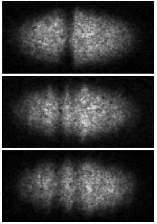
[Billy et al., Nature (2008)]

Nonlinear dispersive shock waves



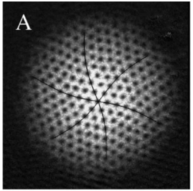
[Hoefer et al., PRA (2006)]

Grey solitons



[Burger et al., PRL (1999)]

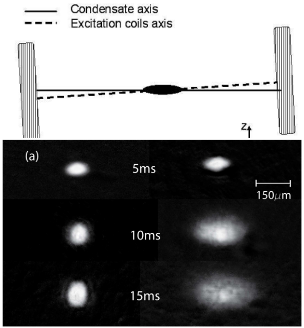
Vortex lattice modes

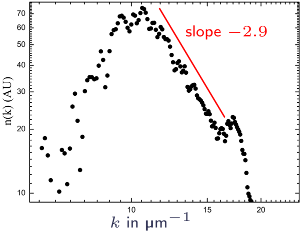


[Coddington et al., PRL (2003)]

Turbulence in BECs (I)

Pioneering work in São Carlos





[Henn et al., PRL (2009), Thompson et al., Laser Phys. Lett. (2014), arXiv:2407.11237]

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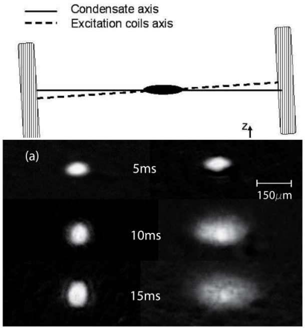
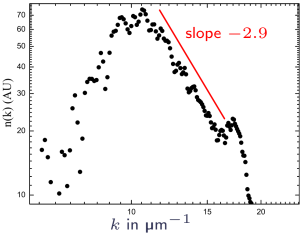
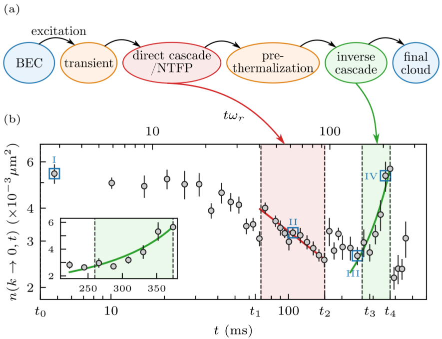




Observation of relaxation stages in a non-equilibrium closed quantum system: decaying turbulence in a trapped superfluid

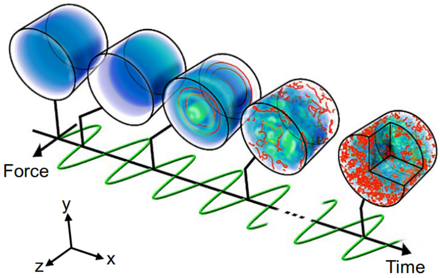
M. A. Moreno-Armijos,[1,*] A. R. Fritsch,[1] A. D. García-Orozco,[1] S. Sab,[1] G. Telles,[1] Y. Zhu,[2] L. Madeira,[1] S. Nazarenko,[2] V. I. Yukalov,[1,3] and V. S. Bagnato[1,4,5]

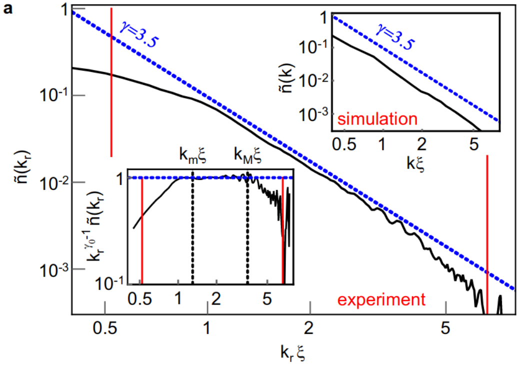


[Henn et al., PRL (2009), Thompson et al., Laser Phys. Lett. (2014), arXiv:2407.11237]

Turbulence in BECs (II)

The Cambridge experiment: turbulence in a box





- ⇒ $\omega_{\mathrm{drive}} = 2\pi \times 9\,\mathrm{Hz}$ & $U_d \sim k_B \times 60\,\mathrm{nK}$
- ⇒ steady-state $n(k)$ for $2\,\mathrm{s} \leq t \leq 4\,\mathrm{s}$
- ⇒ consistent with numerical simulations

Similar experiment in a 2D square box $\gamma = 2.9$

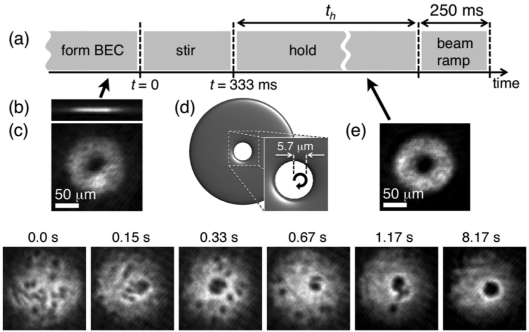
Contribution of waves versus vortices in this scenario ?

[Navon et al., Nature (2016), Gałka et al., PRL (2022)]

[Recent preprint on inverse cascade: arXiv:2405.01537]

Vortex turbulence ?

Hard to see a 3D vortex tangle in experiments: 2D is better ⇒ test Onsager's predictions



⇒ random initial vortex distribution

⇒ emergence of large scale flow

[Neely et al., PRL (2013)]

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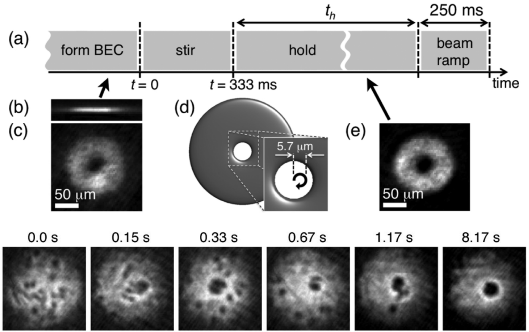

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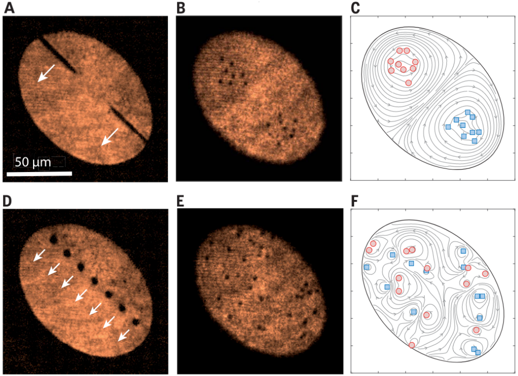


⇒ control over the vortex injection

[Gauthier et al., Science (2019)]

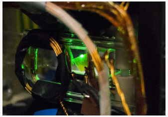
The BEC group at Villetaneuse

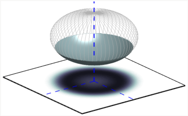
Laboratoire de Physique des Lasers

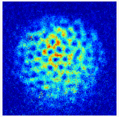
H. Perrin

http://bec.lpl.univ-paris13.fr

Bubble trap experiment — Rb

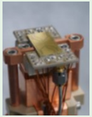


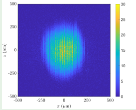


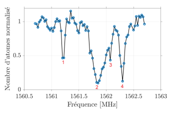


Thermal melting of a vortex lattice
arXiv:2404.05460

Atomchip experiment — Na

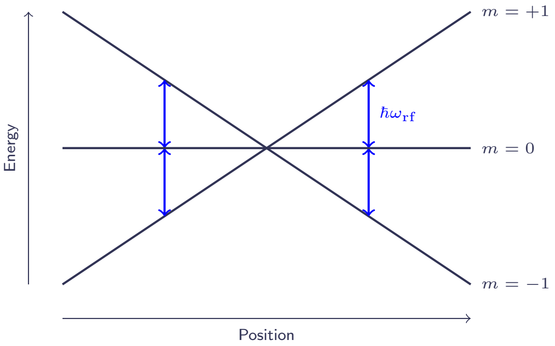






Fast manipulation of a quantum gas on an atom chip with a strong microwave field
arXiv:2405.07583

Main interest: study quantum gases dynamics in low dimensions quadrupole field: $\boldsymbol{B}_0 = b'(x\boldsymbol{e}_x + y\boldsymbol{e}_y - 2z\boldsymbol{e}_z)$ & rf photons

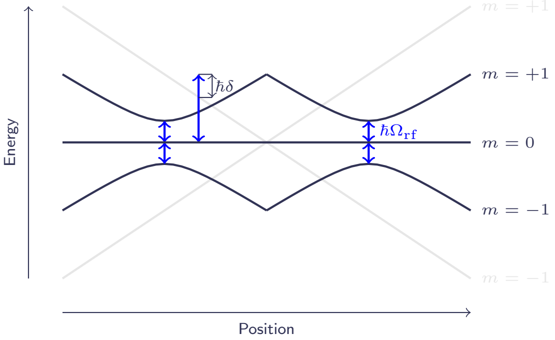


Energy levels of a groundstate ^{87}Rb atom $(F = 1)$

Bubble trap

LPL Laboratoire de Physique des Lasers

quadrupole field: $\boldsymbol{B}_0 = b'(x\boldsymbol{e}_x + y\boldsymbol{e}_y - 2z\boldsymbol{e}_z)$ & rf photons



Energy levels of a groundstate ^{87}Rb atom $(F=1)$

The atom-field coupling results in an energy minimum on the **isomagnetic surface** $\hbar\omega_{\mathrm{rf}} = \mu|\boldsymbol{B}_0|$.

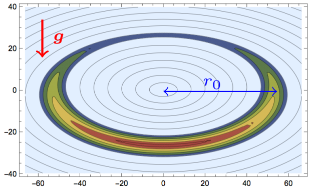
[Zobay & Garraway PRL (2001)]

Adiabatic potential: $V(\boldsymbol{r}) = \hbar\sqrt{\delta(\boldsymbol{r})^2 + \Omega_{\mathrm{rf}}(\boldsymbol{r})^2}$

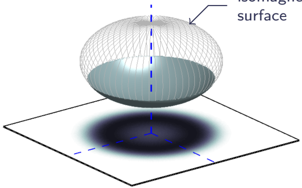
$\delta(\boldsymbol{r}) = \omega_{\mathrm{rf}} - \mu|\boldsymbol{B}_0|/\hbar$

Superfluid on a curved space

side view (isopotentials)



The motion of the atoms is constrained on a 2D curved surface "bubble geometry".



Superfluid on a curved space

$$\Omega_{\mathrm{rf}} \sim 20 - 100\,\mathrm{kHz}$$
$$r_0 \propto \omega_{\mathrm{rf}}/b' \sim 20 - 200\,\mu\mathrm{m}$$
$$\omega_{x,y} \propto \sqrt{g/r_0} \sim 20 - 50\,\mathrm{Hz}$$
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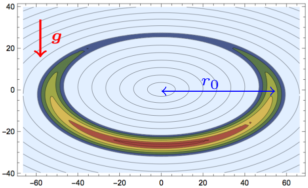

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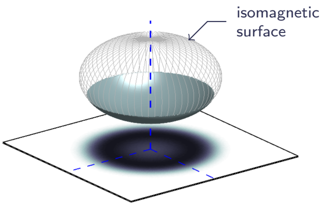
$\Rightarrow$ very thin $\omega_z \gg \omega_{x,y}$



Superfluid on a curved space

$$\begin{aligned} \Omega_{\mathrm{rf}} &\sim 20 - 100\,\mathrm{kHz} \\ r_0 &\propto \omega_{\mathrm{rf}}/b' \sim 20 - 200\,\mu\mathrm{m} \\ \omega_{x,y} &\propto \sqrt{g/r_0} \sim 20 - 50\,\mathrm{Hz} \\ \omega_z &\propto b'/\sqrt{\Omega_{\mathrm{rf}}} \sim 0.3 - 2\,\mathrm{kHz} \end{aligned}$$

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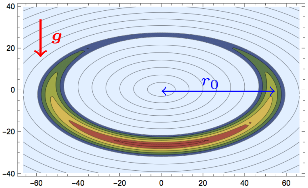
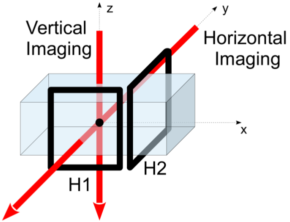


The motion of the atoms is constrained on a 2D curved surface “bubble geometry”.

- $\Rightarrow$ very thin $\omega_z \gg \omega_{x,y}$
- $\Rightarrow$ rf polarization controls the ω_x / ω_y anisotropy
  - rotationally invariant ($\omega_x = \omega_y$) for σ^+
  - anisotropic ($\omega_x \neq \omega_y$) in general



Superfluid on a curved space

$$
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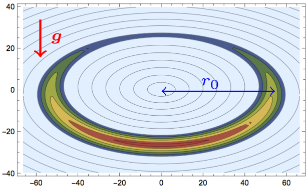

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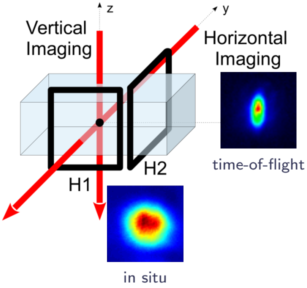
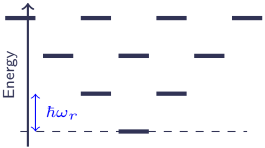
We measure the cloud properties by comparing **pictures of the density** profiles to models.



Motivation for reaching the *fast rotation* regime

Rotation in a 2D harmonic trap: simulation of quantum Hall physics



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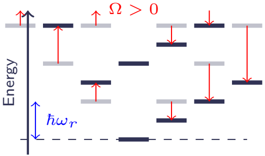

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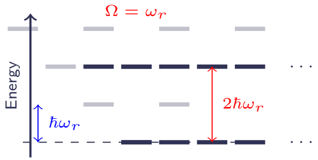

Landau levels hamiltonian

[Schweikhard et al. PRL (2004),
Fletcher et al. Science (2021)]

When $\Omega = \omega_r$: the centrifugal force **cancels** the trapping force

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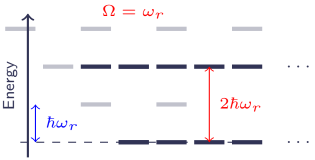

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Add a quartic term : stabilizes the cloud

$$V(r) = \frac{M\omega_r^2}{2} r^2 (1 + \kappa r^2)$$

[Bretin et al. PRL (2004)]

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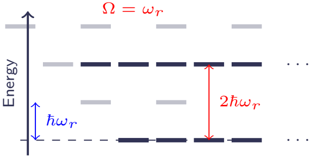

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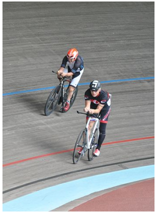

omnium, wikimedia

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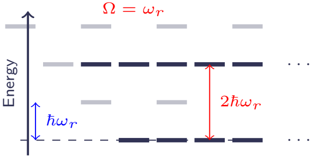

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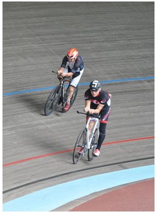
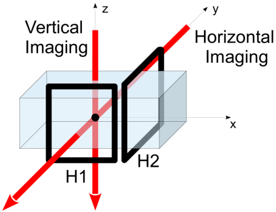
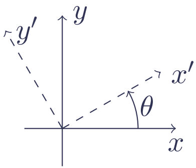


omnium, wikimedia

Highly degenerate groundstate means also higher sensitivity to fluctuations !

How to rotate ?





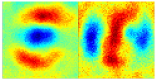
⇒ use a weakly elliptic rf polarization

⇒ angle & amplitude are fully controlled $\theta(t) = \Omega_{\text{stir}} t.$

$$V_{\text{trap}} \simeq \frac{M}{2}\omega_r^2 \left[(1+\epsilon)x'^2 + (1-\epsilon)y'^2\right]$$

⇒ couples to the BEC quadrupole mode

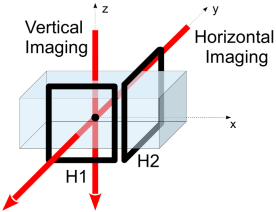
⇒ resonant coupling for:

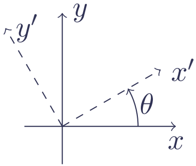


$$\Omega_{\text{stir}} = \frac{\omega_r}{\sqrt{2}} \simeq 2\pi \times 24 \text{ Hz}$$

[Chevy et al. PRL 2000, Abo-Shaeer et al. Science 2001]

How to rotate ?





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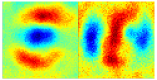
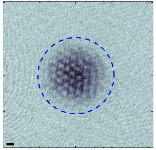


[Chevy et al. PRL 2000, Abo-Shaeer et al. Science 2001]

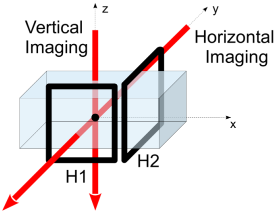
Increasing $\Omega_{\rm rot}$...

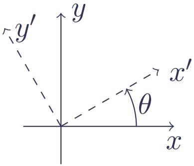


$\Omega_{\rm rot}/(2\pi) = 20$ Hz

Blue dashed circle: Thomas-Fermi radius after 27 ms time-of-flight $\qquad \epsilon = 0.18$

How to rotate ?





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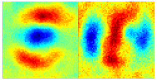
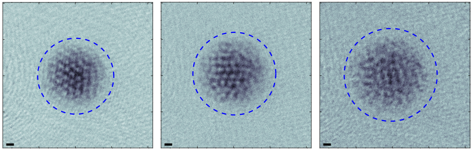


[Chevy et al. PRL 2000, Abo-Shaeer et al. Science 2001]

Increasing Ω_{rot}... disordered lattice...

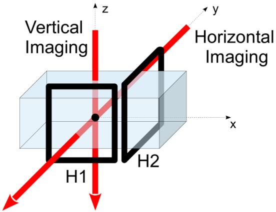


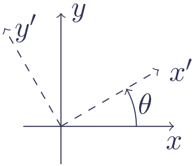
$\Omega_{\text{rot}}/(2\pi) = 20$ Hz | 21 Hz | 24.5 Hz

Blue dashed circle: Thomas-Fermi radius after 27 ms time-of-flight

$\epsilon = 0.18$

How to rotate ?





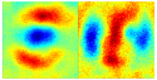
⇒ use a weakly elliptic rf polarization

⇒ angle & amplitude are fully controlled $\theta(t) = \Omega_{\text{stir}} t.$

$$V_{\text{trap}} \simeq \frac{M}{2}\omega_r^2 \left[(1+\epsilon)x'^2 + (1-\epsilon)y'^2\right]$$

⇒ couples to the BEC quadrupole mode

⇒ resonant coupling for:



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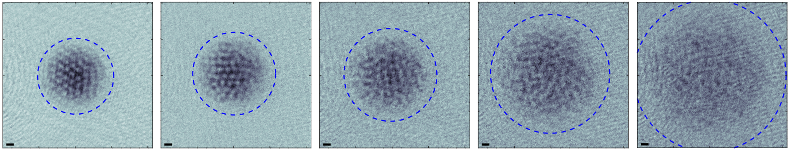
Increasing Ω_{rot}... disordered lattice... melting ?

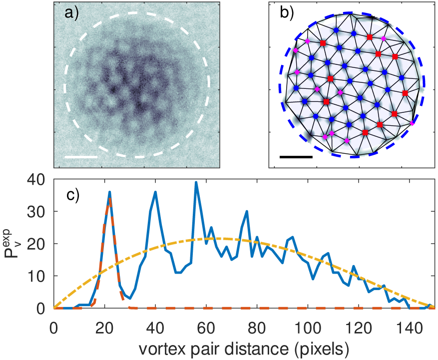


$\Omega_{\text{rot}}/(2\pi) = 20$ Hz | 21 Hz | 24.5 Hz | 28.5 Hz | 30 Hz

Blue dashed circle: Thomas-Fermi radius after 27 ms time-of-flight $\epsilon = 0.18$

Thermal melting of a vortex lattice

arXiv:2404.05460



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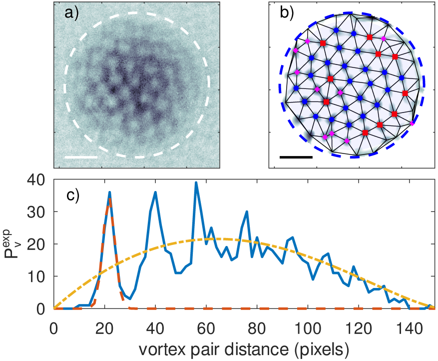

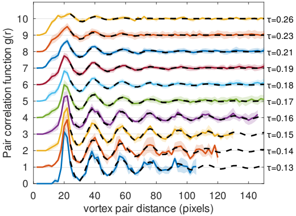

Study vortex lattice order:

$\Rightarrow$ pair correlation function decay $g(r)$

[Sharma et al., arXiv:2404.05460, accepted in PRL]

Thermal melting of a vortex lattice
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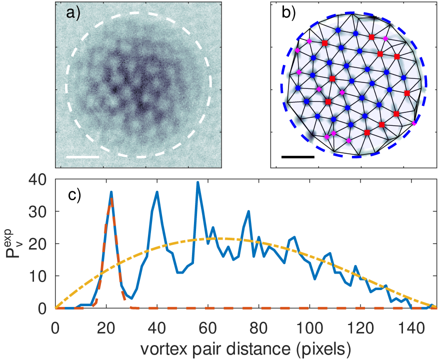

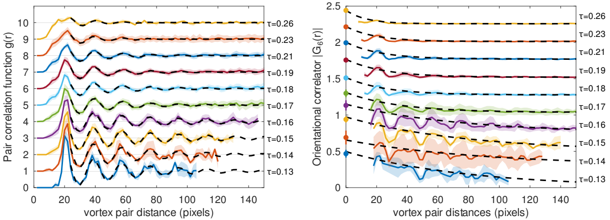

Study vortex lattice order:

- ⇒ pair correlation function decay $g(r)$
- ⇒ lattice order parameter: $\psi_6(\boldsymbol{r}_k) = \frac{1}{N_v}\sum_{j=1}^{N_v} e^{i6\theta_{kj}}$
- ⇒ orientational correlator

$$G_6(r) = \langle \psi_6(\boldsymbol{r}_k)^* \psi_6(\boldsymbol{r}_p) \rangle_{|\boldsymbol{r}_k - \boldsymbol{r}_p| \sim r}$$

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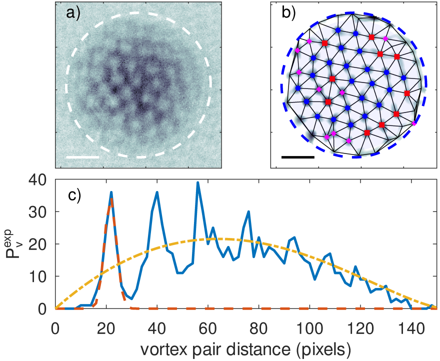

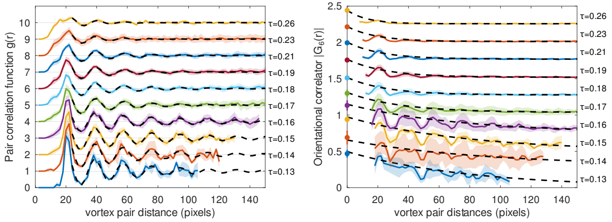

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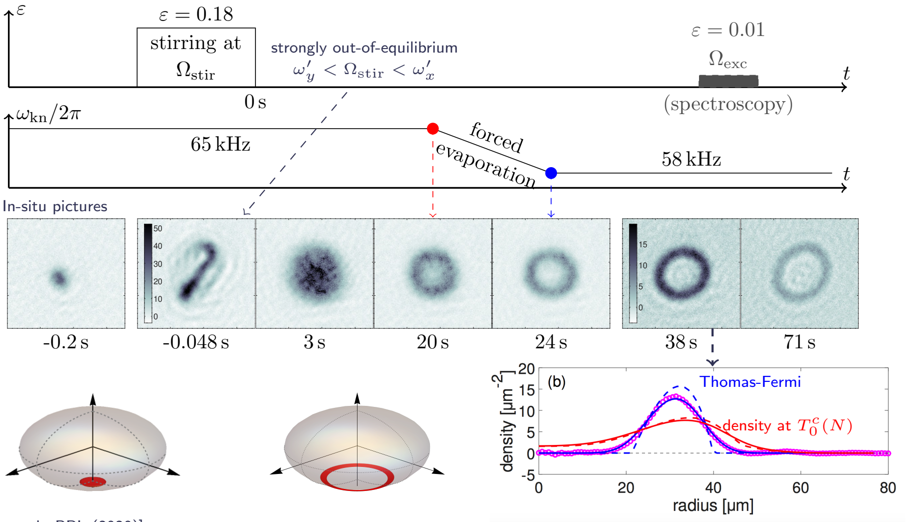
We evidence a transition from a hexatic phase to a liquid (of vortices)

[Sharma et al., arXiv:2404.05460, accepted in PRL]

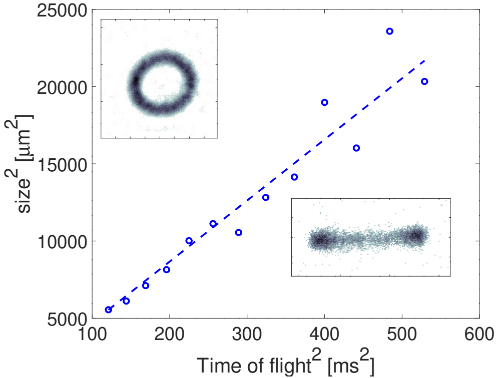
[Well known phenomenon in 2D melting: colloids, dusty plasmas, ...]

Rotating faster: the dynamical ring regime



[Guo et al., PRL (2020)]

A supersonic flow



- $\Rightarrow$ size^2 scales as t^2_{ToF} (ballistic expansion)
- $\Rightarrow$ fit gives: $\Omega \sim 1.05\omega_r$ i.e. $v = 7.4\,\mathrm{mm/s}$
- $\Rightarrow$ peak density is $n_0 \sim 15\,\mu\mathrm{m}^{-2} \Rightarrow$ $c_0 \sim 0.4\,\mathrm{mm/s}$

A degenerate gaz flowing at Mach 18 !

[Guo et al., PRL (2020)]

Gravity compensation

For atoms located on the resonant surface $\delta(\boldsymbol{r}) \simeq 0$,
the trap potential becomes very simple:

$$V(\boldsymbol{r}) = \hbar\sqrt{\delta(\boldsymbol{r})^2 + \Omega_{\rm rf}(\boldsymbol{r})^2} + Mgz \quad \Rightarrow \quad V(\boldsymbol{r}) \simeq \frac{\hbar\Omega_0}{2} + \left(Mg - \frac{\hbar\Omega_0}{r_0}\right) z$$

(for a circular σ^+ rf polarization)

We may define an *effective gravity*:

$$g_{\rm eff} = g\left(1 - \frac{\hbar\Omega_0}{Mgr_0}\right)$$

where $r_0 = \omega_{\rm rf}/\alpha \propto \omega_{\rm rf}/b'$

Changing r_0 enables a compensation of the gravitational potential...

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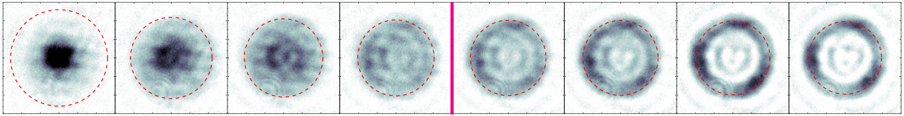
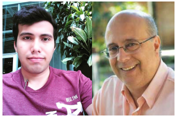




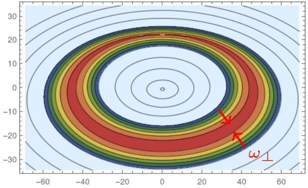
E. Mercado, V. Bagnato

[Guo et al. NJP (2022)]

Why do we see a **ring** ?

The unexpected contribution of the third dimension

isopotentials for $g_{\rm eff} \simeq 0$

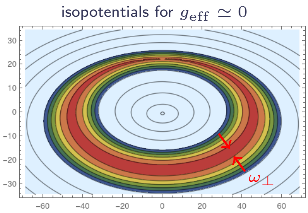


$$V(\boldsymbol{r}) = \frac{\hbar\Omega_0}{2} + M g_{\rm eff} z + \frac{\hbar\omega_\perp(z)}{2} \qquad \omega_\perp(z) \simeq \alpha(z)\sqrt{\frac{\hbar}{M\Omega(z)}}$$

Confinement to the surface increases near the top:

atoms are repelled $\Rightarrow$ stable ring shape.

The unexpected contribution of the third dimension



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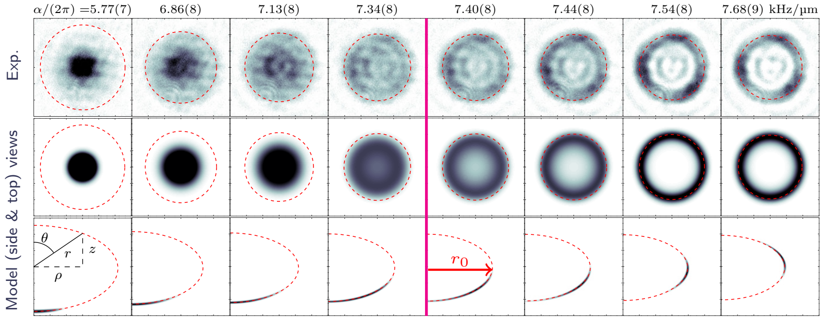


Modelisation

Using:

⇒ a refined $V(\boldsymbol{r})$

Floquet expansion

⇒ $T = 0$ superfluid density model

(Gross-Pitaevskii equation)

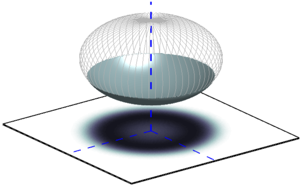
[Guo et al. NJP (2022)]

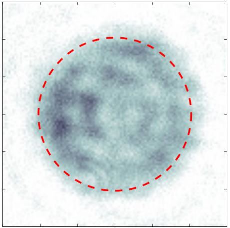
Project VORTECS

Vortex turbulence on a curved surface

So we have a curved superfluid, should we expect new physics ?

$\Rightarrow$ yes, in a **rotating frame**: β-effect, Rossby waves, zonal flows, ... even turbulence ?





We aim for:

- $\Rightarrow$ a large area 2D superfluid $d \gg \xi$

  $d \sim 100\,\mu\text{m}$, $\xi \sim 1\,\mu\text{m}$
- $\Rightarrow$ a very low temperature $T \leq 20\,\text{nK}$
- $\Rightarrow$ high imaging resolution $< 0.5\,\mu\text{m}$

Collaboration with Ying, Simon, Éric & Sergey

Summary
Take home messages

- ultracold atom platforms are interesting to study nonlinear physics
  with a *bottom-up* approach

⇒ some experiments are really close to implementing **textbook models**

- we do need intensive **numerical simulations** to help guide the experiments
  especially *far from equilibrium*

- at LPL we have a unique platform to study rotating superfluids in a curved space

Acknowledgements

front: M. Cassus, B. Mirmand, M. Ballu, L. Longchambon, R.D., R. Sharma, S. Bereta

behind: A. Perrin, H. Perrin, T. Badr

M. De Goër, Y. Guo, D. Rey

Thank you for your attention !